Integration Practice Problems

Perfect Math Tactics

June 6 2024

Hints on the last page. Problems are labeled with difficulty on a scale of 1-10.

1. (6) Compute the indefinite integral

$$\int \frac{e^{2x}dx}{(e^x+1)(e^x+3)}$$

2. (7) Compute the indefinite integral

$$\int \frac{\sin^4 x}{\cos^2 x}dx$$

3. (6) Compute the indefinite integral

$$\int \frac{\sin^3 x}{\cos x}dx$$

4. (9) Compute the indefinite integral

$$\int \frac{\sin^2 x}{\cos x}dx$$

5. (5) Compute the integral

$$\int_1^{e^2} (\ln x)^2 dx$$

6. (7) Compute the indefinite integral

$$\int x^3\cos(x^2)dx$$

7. (8) Compute the indefinite integral

$$\int xe^{\sqrt{x}}dx$$

8. (7) Compute the indefinite integral

$$\int e^{x^{1/3}}dx$$

   Any similiarities with the previous problem?

9. (7) Compute the improper integral

$$\int_0^\infty \frac{2x+3}{(x^2+9)(x+1)}dx$$

   Solve the same problem with bounds replaced by -2 to 0 and -1 to 1.

10. (5) Compute the arc length of $f(x) = x^5 + \frac{1}{60x^3}$ on the interval $[1, 3]$. Try to come up with variants of this kind of function for arc length.

11. (6) Compute the arc length of $f(t) = \frac{e^t + e^{-t}}{2}$ on the interval $[-1, 1]$.

12. (9) Compute the arc length on the parabola $y = x^2/2$ from $(2, 2)$ to $(4, 8)$.

Hints

1. Do a u-substitution and partial fraction decomposition (PFD).
2. Use the identity $\sin^2 x + \cos^2 x = 1$ to rewrite the numerator. You may need to split the result into several integrals.
3. Use a u-substitution along with a trig identity.
4. (Probably too hard for an exam) Multiply the numerator and denominator by $\cos x$. Then apply a trig identity to rewrite and factor the denominator, and use u-substitution along with PFD.
5. You will need to apply integration by parts twice.
6. Perform a u-substitution and then integrate by parts. Usually, the u-substitution partially simplifies the integral.
7. Let $t = \sqrt{x}$, which means $x = t^2$. Write dx in terms of dt. You may need tabular integration.
8. Let $t = x^{1/3}$. Same hint as previous problem.
9. An expression of the form $A \ln(x + c)$ can be written as $\frac{A}{2} \log(x + c)^2$. This should help with the $\int_0^\infty$ case. For the $\int_{-2}^0$ case, split the integral across a vertical asymptote (you should see integrals approaching different infinities). For the $\int_{-1}^1$ case, you should get another infinite integral.
10. the expression $1 + f'(x)^2$ should factor as a perfect square.
11. Same hint as previous problem.
12. You will need the substitution $x = \tan\theta$ (along with cyclic integration) to integrate $\sqrt{1 + x^2}$. Break up the resulting integral into $u = \sec\theta, dv = \sec^2\theta$. Use a trig identity to rewrite the vdu integral, and you should see the original integral appear again.